HELP YOUR STUDENTS DISCOVER A PROOF

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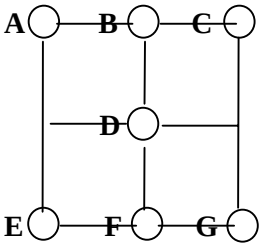
The most striking aspect of mathematical knowledge is that it is obtained from mathematical proofs. Statements in mathematics that have been accepted as true, are those which have been proven. The concept of proof in mathematics is of paramount importance.

The reason why students have such great difficulties and anxiety when asked to construct a proof, is because they have had so little experience constructing proofs.

The following is offered as one way teachers may involve their students in the comprehension, construction and discovery of a proof.



Is it possible to place counting numbers in the circles so that each line has the same total?

After small group work in which the students are provided an opportunity to construct a proof independently of the teacher's help, the teacher than leads the students through a proof, such as the following:

We know that all the numbers that go in the circles where A,B,C,D,E,F, and G are, must be greater than zero.

Then, A + B + C = some number, N.
E + F + G = N
A + D + G = N

Adding both sides:

$$\mathbf{2A + B + C + E + F + 2G + D = 3N} \qquad \mathbf{(i)}$$

Also, A + E = N
C + G = N
B + D + F = N

So, $\mathbf{A + B + C + E + F + G + D = 3N}$ **(ii)**

Subtracting (ii) from (i) implies A + G = 0, which is impossible. Q.E.D.

After having "led" the students through this proof, ask them to find another proof similar to the previous one. The teacher may wish to erase the circle with the "D" in it, if they have not "discovered" something similar to the following:

A,B,C,E,F,G greater than 0.

Then $A + B + C = N$
$E + F + G = N$

So $A + B + C + E + F + G = 2N$

Also, $A + E = N$
$C + G = N$

So $A + C + E + G = 2N$

Implies $B + F = 0$, which is not possible. Q.E.D.